Multicomponent binary spreading process

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I investigate numerically the phase transitions of two-component generalizations of binary spreading processes in one dimension. In these models pair annihilation $AA \to \emptyset$, $BB \to \emptyset$, explicit particle diffusion, and binary pair production processes compete with each other. Several versions with spatially different production are explored, and it is shown that for the cases $2A \to 3A$, $2B \to 3B$ and $2A \to 2AB$, $2B \to 2BA$ a phase transition occurs at zero production rate $(\sigma=0)$, which belongs to the class of N-component, asymmetric branching and annihilating random walks, characterized by the order parameter exponent $\beta=2$. In the model with particle production $AB \to ABA$, $BA \to BAB$ a phase transition point can be located at $\sigma_c=0.3253$ which belongs to the class of one-component binary spreading processes.

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One-dimensional, nonequilibrium phase transitions have been found to belong to a few universality classes, the most robust of them being the directed percolation (DP) class [1,2]. According to the hypothesis of [3,4] all continuous phase transitions to single absorbing states in homogeneous, single component systems with short ranged interactions belong to this class, provided there is no additional symmetry or quenched randomness present. The best known exception to the robust DP class is the parity conserving (PC) class [5], where a mod 2 conservation of particles happens [for example, in even offspring branching and annihilating random walks (BARWE)] and in multiabsorbing state systems where an exact Z_2 symmetry is also satisfied [6]. There are other classes being explored recently where the total number of particles is conserved [7–13].

In multicomponent systems bosonic field theory [14], simulations [15], and density matrix renormalization group analysis [16] have revealed the universality class of the generalization of the BARWE class. Hard-core particle exclusion effects can change both the dynamic [17,18] and static [15,19–22] behavior of one-dimensional systems by introducing blockades into the particle dynamics. Earlier it was shown that an infinite number of conservation laws emerge in stochastic deposition-evaporation models of Q-mers in one dimension [23,24] that split up the phase space into kinetically disconnected sectors. That results in initial-condition-dependent autocorrelation functions.

In [18] a two-component generalization of the annihilating random walk (2-ARW) model was introduced taking into account hard-core repulsion of particles:

$$AA \xrightarrow{\lambda} \emptyset, \quad BB \xrightarrow{\lambda} \emptyset, \quad A\emptyset \overset{d}{\leftrightarrow} \emptyset A, \quad B\emptyset \overset{d}{\leftrightarrow} \emptyset B, \quad AB \not\leftrightarrow BA \qquad (1)$$

(where λ and d denote the annihilation and diffusion rates) and it was shown that the initial conditions influence the particle density (order parameter) decay and the dynamical exponents. On adding pair creation processes $(A \xrightarrow{\sigma} 2BA, B \xrightarrow{\sigma} 2AB)$ to this model, a continuous phase transition occurs at creation rate $\sigma=0$ and two universality classes appear depending on the arrangement of the offspring relative to the parent [15]; namely, if the parent separates the two offspring $A \xrightarrow{\sigma} BAB$ (2-BARW2S) (symmetric) the steady state density will be higher than in the case when they are created on the same site, $A \xrightarrow{\sigma} ABB$ (2-BRAW 2A) (antisymmetric) for a given branching rate σ because in the former case they are unable to annihilate each other. This results in different off-critical order parameter exponents for the symmetric and asymmetric cases ($\beta_s=1/2$ and $\beta_a=2$). This is in contrast to the widespread belief that bosonic field theory can well describe reaction-diffusion systems in general. In the field theoretical version [14], where the $AB \leftrightarrow BA$ exchange is allowed, the critical behavior is different. Mean-field-like and simulation results led Kwon *et al.* [19] to the assumption that in one-dimensional reaction-diffusion systems a series of new universality classes should appear if particle exclusion is present.

In a recent paper [21] I showed that if one adds single particle creation to the 2-ARW model,

$$A \xrightarrow{\sigma} AB, \quad B \xrightarrow{\sigma} BA, \qquad (2)$$

a continuous phase transition occurs again at $\sigma=0$ and the critical exponents coincide with that of the 2-BARW2S model, although the parity of the particle number is not conserved. Therefore, this conservation law, which was relevant in the case of one-component systems (PC versus DP class), is irrelevant here. In [21] I made the hypothesis that in coupled branching and annihilating random walk systems of N types of excluding particles with continuous transitions at $\sigma=0$, two universality classes exist, those of the 2-BARW2S and 2-BARW2A models, depending on whether the reactants can immediately annihilate [i.e., when similar particles are not separated by other type(s) of particle(s)] or not. These classes differ only in the off-critical exponents, while the on-critical ones are the same. This is due to the fact that the critical point is at zero branching rate $(\sigma=0)$ and therefore the critical exponents are the ones determined for the 2-ARW model [18,15].

In this paper I extend the investigation to coupled binary production spreading processes, where new universal behavior has recently been reported. Studies on the annihilation fission process $2A \to \emptyset$, $2A \to 3A$, $A\emptyset \leftrightarrow \emptyset A$ [25–29] found evidence that there is a phase transition in this model that does not belong to any previously known universality class. This model without the single particle diffusion term—the so called pair contact process (PCP), where pairs of particles can annihilate or create new pairs—was introduced originally by Jensen [32], and while the static exponents were found to belong to the DP class the spreading ones show nonuniversal behavior. By adding explicit single particle diffusion [26] Carlon *et al.* introduced the so called PCPD particle model. The renormalization group analysis of the corresponding bosonic field theory was given by Howard and Täuber [25]. This study predicted a non-DP class transition, but it could not tell to which universality class this transition really belongs. An explanation based on symmetry arguments is still missing but numerical simulations suggest [28,33] that the behavior of this system can be well described (at least for strong diffusion) by coupled subsystems: single particles performing annihilating random walks coupled to pairs (B) following a DP process: $B \to 2B$, $B \to \emptyset$. The model has two nonsymmetric absorbing states: one is completely empty and in the other a single particle walks randomly. Owing to this fluctuating absorbing state this model does not oppose the conditions of the DP hypothesis.

In the low diffusion region ($d < \sim 0.4$) some exponents of the PCPD model are close to those of the PC class but the order parameter exponent (β) has been found to be very far away from the values of both the DP and the PC class [28]. In fact, this system does not exhibit either the Z_2 symmetry or the parity conservation which appear in models with a PC class transition. In the high diffusion region the critical exponents seem to be different [26,28,30], suggesting another universality class there [28]. This is also supported by the pair mean-field results [26]. A recent universal finite size scaling amplitude study [31] suggests, however, that a single universality class with strong corrections to scaling may also be possible.

It is conjectured by Henkel and Hinrichsen [34] that this kind of phase transition appears in models where (i) solitary particles diffuse, (ii) particle creation requires two particles, and (iii) particle removal requires at least two particles to meet. Very recently, Park *et al.* [35] have investigated the parity conserving version of the PCPD model ($2A \to 4A, 2A \to \emptyset, A\emptyset \leftrightarrow \emptyset A$) and, contrary to the apparent conservation law, they found similar scaling behavior, which led them to the assumption that the binary nature of the offspring production is a necessary condition for this class. Other conditions that would influence the occurrence of this class should be clarified too. In this paper I address the question of whether the particle exclusion effects are relevant, as in the case of BARW processes, and whether the hypothesis set up for N-BARW systems [21] could be extended.

One site update step of the applied algorithms consists of the following processes. A particle is selected randomly. A left or right nearest neighbor is chosen with probability 0.5. With probability σ pair production is attempted in the case of an appropriate neighbor. Otherwise (with probability $d = \lambda = 1 - \sigma$) hopping is attempted if the neighboring site is empty, or if it is filled with a particle of the same type they are annihilated. The following models with the same diffusion and annihilation terms as Eq. (1) and different production processes will be investigated here.

(a) The production and annihilation random walk model (2-PARW):

$$AA \xrightarrow{\sigma/2} AAB, \quad AA \xrightarrow{\sigma/2} BAA, \tag{3}$$

$$BB \xrightarrow{\sigma/2} BBA, \quad BB \xrightarrow{\sigma/2} ABB \tag{4}$$

(b) The symmetric production and annihilation random walk model (2-PARWS):

$$AA \xrightarrow{\sigma} AAA, \tag{5}$$

$$BB \xrightarrow{\sigma} BBB. \tag{6}$$

(c) The asymmetric production and annihilation random walk model (2-PARWA):

$$AB \xrightarrow{\sigma/2} ABB, \quad AB \xrightarrow{\sigma/2} AAB, \tag{7}$$

$$BA \xrightarrow{\sigma/2} BAA, \quad BA \xrightarrow{\sigma/2} BBA. \tag{8}$$

(d) The asymmetric production and annihilation random walk model with spatially symmetric creation (2-PARWAS):

$$AB \xrightarrow{\sigma/2} ABA, \quad AB \xrightarrow{\sigma/2} BAB, \tag{9}$$

$$BA \xrightarrow{\sigma/2} BAB, \quad BA \xrightarrow{\sigma/2} ABA. \tag{10}$$

The evolution of particle densities was followed by Monte Carlo simulations started from randomly distributed $A,B,\emptyset$ sites in systems of sizes $L = 10^5$ and with periodic boundary conditions.

The 2-PARWA model (c) does not have an active steady state. The AA and BB pairs annihilate themselves on contact, while if an A and B particle meet an $AB \to ABB \to A$ process reduces blockades, so the densities decay with a $\rho \propto t^{-1/2}$ law for $\sigma > 0$. This was confirmed by my simulations. Note that for $\sigma = 0$ the blockades persist and in the case of a random initial state a $\rho \propto t^{-1/4}$ decay can be observed [15].

The 2-PARW (a) and 2-PARWS (b) models exhibit active steady states for $\sigma > 0$ with a continuous phase transition at $\sigma = 0$. Therefore the exponents at the critical point will be those of the ARW-2 model. The convergence to the steady state is very slow. For $\sigma = 0.1$ it was longer than 10^9 Monte Carlo steps (MCS). This limits the simulations in approaching the critical point at $\sigma = 0$. However, as Fig. 1 shows, a rather good scaling behavior of the density versus σ can be observed.

The local slope analysis shows that the scaling behavior extrapolates to $\beta = 2.1(2)$ in the 2-PARWS model and to $\beta = 1.9(2)$ in the 2-PARW model. These values are in agreement with those of the 2-BARW2A class ($\beta = 2$), where production is such that pair annihilation is enhanced.

In the case of the 2-PARWAS model (d) the AB blockades proliferate by production events. As a consequence of this an

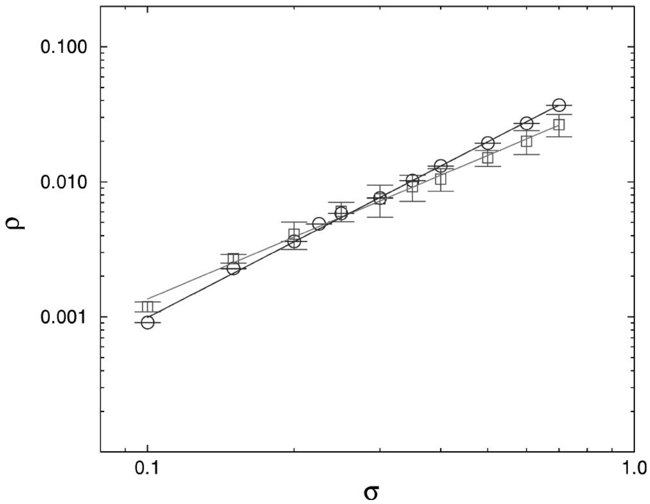

FIG. 1. Steady state densities as a function of σ in the 2-PARW (squares) and 2-PARWS (circles) models.

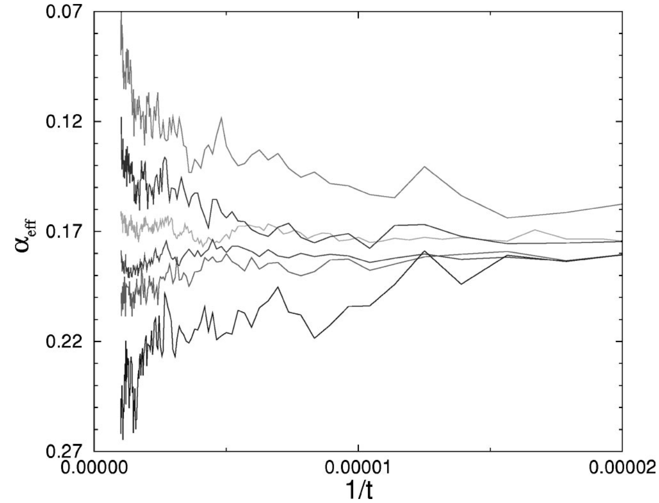

FIG. 3. Local slopes of the density decay in the PARWAS model. Different curves correspond to $\sigma = 0.325$, 0.3252, 0.3253, 0.3254, 0.3255, and 0.326 (from bottom to top).

active steady state appears for $\sigma > 0.3253(1)$ with a continuous phase transition. The space-time evolution from a random initial state shows (Fig. 2) that compact domains of alternating ...*ABAB*... sequences separated by lonely wandering particles are formed. This is very similar to what was seen in the case of one-component binary spreading processes [33]: compact domains within a cloud of lonely random walkers, except that now domains are built up from alternating sequences only. This means that ...*AAAA*... and ...*BBBB*... domains decay at this annihilation rate and particle blocking is responsible for the compact clusters. In the language of the coupled DP+ARW model [33], the pairs following the DP process are now the *AB* pairs, which cannot decay spontaneously but through an annihilation process: $AB + BA \rightarrow \emptyset$. They interact with two types of particle executing annihilating random walks with exclusion.

Simulations from random initial states were run for up to 10^6 MCS. The local slopes of the particle density decay

$$\alpha_{\mathrm{eff}}(t) = \frac{-\ln[\rho(t)/\rho(t/m)]}{\ln(m)} \tag{11}$$

(where $m = 8$ is used) at the critical point approach the exponent α asymptotically as a straight line, while in sub-(super)critical cases they veer down (up) (see Fig. 3). At the critical point $[\sigma_c = 0.3253(1)]$ one can estimate that the effective exponent tends to $\alpha = 0.19(1)$, which is higher than the exponent of $(1+1)$-dimensional directed percolation [0.1595(1)] [36] and in fairly good agreement with that of the PCPD model in the high diffusion rate region [0.20(1)] [28].

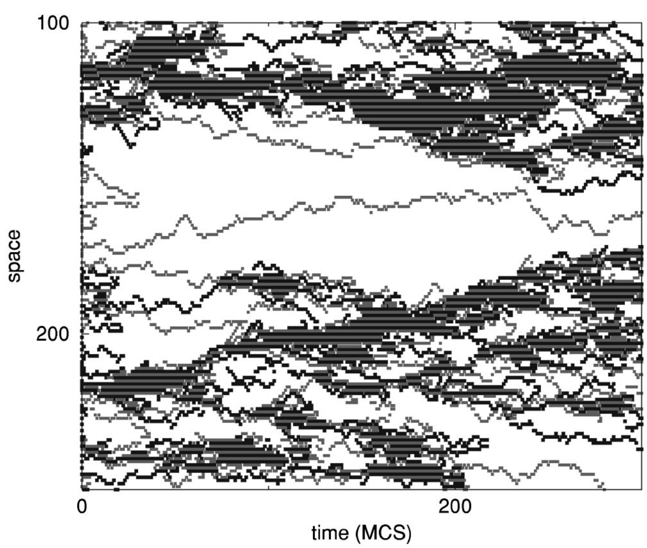

FIG. 2. Space-time evolution from random initial state of the 2-PARWAS model at the critical point. Black dots correspond to *A* particles, gray dots to *B*'s.

In the supercritical region the steady states were determined for different $\epsilon = \sigma - \sigma_c$ values. Following level-off the densities were averaged over 10^4 MCS and 1000 samples. By looking at the effective exponent defined as

$$\beta_{\mathrm{eff}}(\epsilon_i) = \frac{\ln \rho(\epsilon_i) - \ln \rho(\epsilon_{i-1})}{\ln \epsilon_i - \ln \epsilon_{i-1}} \tag{12}$$

(Fig. 4) one can read off $\beta_{\mathrm{eff}} \rightarrow \beta \simeq 0.37(2)$, which is again higher than the $(1+1)$-dimensional DP value 0.27649(4) [37], and agrees with that of the PCPD model in the high diffusion rate region [0.39(2)] [28].

Finally, the survival probability $[P(t)]$ of systems started from random initial condition was measured for sizes $L = 50,100,200,400,800,1600$. The characteristic time $\tau(L)$ to decay to $P(\tau) = 0.9$ was determined and is shown on Fig. 5. At criticality one expects the finite size scaling

$$\tau(L) \propto L^Z, \tag{13}$$

where Z is the dynamical exponent. Power-law fitting resulted in $Z = 1.81(2)$, which is far away from the DP value $Z = 1.580\,740\,(34)$ [36] but close to various estimates for the PCPD value $Z = 1.75(10)$ [26,27].

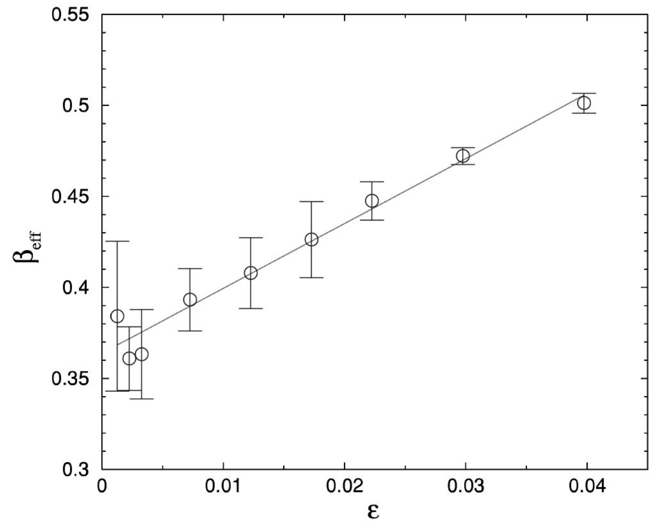

FIG. 4. Effective order parameter exponent results. Linear extrapolation results in $\beta = 0.37(2)$.

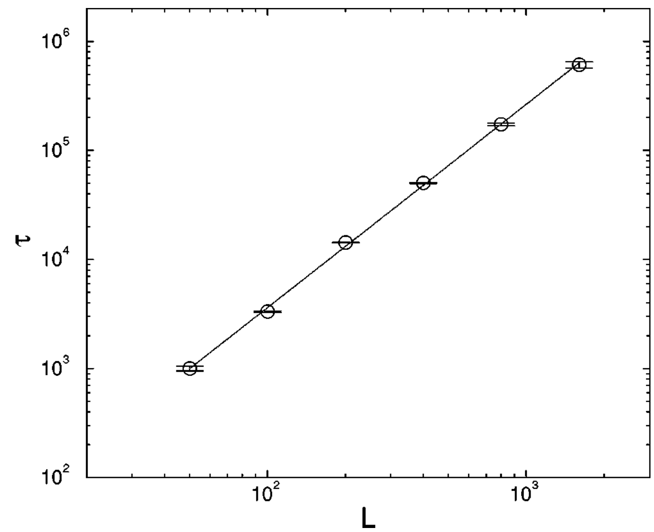

FIG. 5. Lifetime versus system size at the critical point.

In conclusion, I have shown in this work that the hypothesis that I made for *N*-BARW models with exclusion [21] may be extended to coupled binary production annihilation models. The critical point in the 2-PARW and 2-PARWS models occurs at $\sigma = 0$ production rate and therefore the on-critical exponents coincide with those of the 2-ARW model. The simulations for the off-critical behavior of the order parameter showed that the transition belongs to the 2-BARW2A class. The robustness of this class is striking, especially in the case of the 2-PARWS model where in principle two copies of PCPD models are superimposed and coupled by the exclusion interaction only.

If the production is generated by different types of particles (*AB*) such that alternating sequences are generated (2-PARWAS model), the space-time evolution will resemble that of the PCPD model with alternating frozen sequences inside the compact domains. This system exhibits a continuous phase transition at $\sigma = 0.3253(1)$ with exponents in fairly good agreement with those of the PCPD model in the high diffusion region. Therefore, the conjecture of [34] may be extended for multicomponent systems if the transition happens at nonzero production rate. In the model where *AB* pairs create offspring in such a way that prompt annihilation is possible, active steady states are not formed for any σ and the density decays without blockades for $\sigma > 0$ as $\rho \propto t^{-0.5}$, but a crossover to the 2-ARW model scaling $\rho \propto t^{-0.25}$ occurs at $\sigma = 0$.

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